

Lecture Notes: Basic Properties of Nuclei

Anupam Deka

Department of Physics, Nabajyoti College, Kalgachia

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1 The Nucleus

1.1 Introduction

In this chapter, we present the foundational concepts of the nucleus, nuclear constituents, and nuclear properties, such as mass, size, density, and angular momentum. To understand the concept of the nucleus, it is worthwhile to review the development of various atomic models.

1.2 Historical Developments

The beginning of nuclear physics traces back to the discovery of radioactivity in 1896 by Henry Becquerel. Three different modes of radioactivity were observed emitting α -particles, β -particles, and γ -rays. Scattering of α -particles with matter ultimately revealed the existence of a nucleus inside the atom.

1.2.1 Dalton's Atomic Hypothesis

In 1803, Dalton proposed an atomic theory of matter with the following postulates:

- All elements consist of discrete particles, called atoms, which cannot be subdivided by any known chemical process.
- All atoms of the same element are identical in all respects, especially in weight, while atoms of different elements differ in weight.
- A compound is formed by the union of atoms of different elements in simple numerical proportions.

1.2.2 Thomson Model of Atom

J.J. Thomson suggested that atoms are simply lumps of matter with positive charge, and electrons are embedded within them. This is popularly known as the *Plum-Pudding model* or *watermelon model*. It was consistent with overall neutrality but failed to explain α -particle scattering.

1.2.3 Rutherford Model of Atom

In 1911, Ernst Rutherford and his co-workers studied how α -particles scattered when passed through a thin gold foil:

1. Most α -particles went straight through without getting deflected.
2. Some suffered small angle deviations.
3. About 1 in 8000 was back-scattered (angles between 90° and 180°).

Rutherford concluded the atom has a large empty space and the positive charge is concentrated in a minute center called the nucleus (radius estimated at 10^{-14} m).

1.3 Constituents of the Nucleus

The nucleus of the atom has a very small size (its volume is 10^{-15} times the volume of the atom). It contains the whole positive charge and practically the whole mass.

1.3.1 Proton–Electron Hypothesis

It was initially proposed that the nucleus consists of A protons and $A - Z$ electrons, surrounded by Z extra-nuclear electrons.

Failure of Proton-Electron Hypothesis:

- **Spin and statistics:** The total spin of nuclei (e.g., ^{14}N) contradicted experimental results. An even number of protons (14) produces integral spin, while an odd number of electrons (7) gives half-integral spin, yielding a half-integral total spin for ^{14}N , making it a fermion. However, experimentally ^{14}N is a boson with integral spin.
- **Binding energy:** As discussed later in Chapter 8, electron being a lepton cannot take part in strong nuclear interactions. Strong interactions bind the nucleons together in the nucleus. If electrons were present in nucleus, the bound state is caused by Coulomb interaction and the binding energy is of the order of

$$E \simeq -\frac{Ze^2}{r}$$

where r is the radius of the nucleus and is given by $1.2 \times 10^{-15} A^{1/3}$ m or $1.2 \times A^{1/3}$ fm (1 fm = 10^{-15} m). Negative sign means that the system is bound. Thus,

$$E = -Z \frac{e^2}{hc} \frac{hc}{r}$$

where

$$\begin{aligned} \frac{e^2}{hc} &= \alpha = \frac{1}{137} \quad (\text{Fine structure constant}) \\ h &= 1.055 \times 10^{-34} \text{ J s} \\ c &= 3 \times 10^8 \text{ m/s} \end{aligned}$$

Thus,

$$\begin{aligned} E &= -Z \times \frac{1}{137} \times \frac{1.055 \times 10^{-34} \times 3 \times 10^8}{1.2 \times 10^{-15} \times A^{1/3}} \text{ J} \\ &\simeq -\frac{1.2 \times Z}{A^{1/3}} \text{ J} \end{aligned}$$

If $A \approx 125$ and $Z \approx 62$, then

$$\begin{aligned} E &= -\frac{1.2 \times 62}{125^{1/3}} \\ &\approx -15 \text{ MeV} \end{aligned}$$

The de Broglie wavelength of the electrons having energy = 15 MeV would be

$$\lambda = \frac{h}{p} = \frac{ch}{E}$$

Here $cp \approx E$ is the energy of the electron in the nucleus

$$\begin{aligned}\lambda &= \frac{3 \times 10^8 \times 1.055 \times 10^{-34}}{15 \times 1.6 \times 10^{-13}} \\ &= 13 \times 10^{-15} \text{ m} \\ &= 13 \text{ fm}\end{aligned}$$

This value is much greater than the size of the nucleus. Hence, electrons cannot exist inside the nucleus as bound electrons.

- **Nuclear magnetic moment:** Experimental magnetic moment is of the order of nuclear magneton μ_N , not Bohr magneton μ_B ($\mu_B \approx 1850\mu_N$).
- **Beta decay:** The continuous energy spectrum of β -decay contradicts the two-body process implied by this hypothesis.

1.3.2 Proton–Neutron Hypothesis

After the discovery of neutrons by Chadwick in 1932, Heisenberg proposed that nuclei are composed of Z protons and $A - Z$ neutrons.

Following facts strongly support the proton-neutron hypothesis:

1. **Spin:** Both protons and neutrons have a spin quantum number of $1/2$. If the number of nucleons is even, the resultant spin is an integral multiple of $\hbar$, matching experiments.
2. **Magnetic moment:** Because there are no electrons inside the nucleus, the magnetic moment should be of the order of a nuclear magneton (μ_N), which is in perfect agreement with experimental observations.
3. **Finite size:** Since the mass of a neutron is approximately equal to the mass of a proton, it is possible for the neutron to reside inside the nucleus according to the uncertainty principle.
4. **Wave mechanical considerations:** According to the uncertainty principle

$$\Delta x \Delta p \geq h$$

where Δx and Δp are the uncertainties in the position and momentum of proton/neutron respectively. Radius of a typical nucleus of mass ~ 200 is of the order of 0.6×10^{-14} m, the uncertainty in position will be $\Delta x = 2R = 1.2 \times 10^{-14}$ m. Therefore, uncertainty in momentum Δp is given by

$$\begin{aligned}\Delta p &\approx \frac{h}{\Delta x} \\ &= \frac{6.6 \times 10^{-34} \text{ J s}}{1.2 \times 10^{-14} \text{ m}} = 5.5 \times 10^{-20} \text{ J s/m}\end{aligned}$$

Let us estimate the energy E of a nucleon in the nucleus using the relativistic relation

$$\begin{aligned}E^2 &= p^2 c^2 + m^2 c^4 \\ &= (5.5 \times 10^{-20})^2 (3 \times 10^8)^2 + (1.67 \times 10^{-27})^2 (3 \times 10^8)^4 \\ &= 2.29 \times 10^{-20}\end{aligned}$$

which gives

$$E = 1.51 \times 10^{-10} \text{ J} \approx 945 \text{ MeV}$$

This value is slightly greater than the rest mass energy of proton, which is about 938 MeV. Hence, the kinetic energy of neutron or proton in the nucleus is of the order of few MeV (i.e. $945 - 938 \approx 7 \text{ MeV}$) and it should be possible for a free proton or neutron to be confined in the nucleus.

5. **Isotopic masses:** It elegantly explains the existence of isotopes; different isotopes of an element simply have the same number of protons but different numbers of neutrons.
6. **Beta decay:** The process of β -decay is perfectly explained by the transformation of a free neutron into a proton ($n \rightarrow p + \beta^- + \bar{\nu}$) and vice versa ($p \rightarrow n + \beta^+ + \nu$).

1.3.3 Terms Associated with the Nucleus

- **Atomic Number (Z):** Number of protons.
 - **Mass Number (A):** Total number of protons and neutrons.
 - **Neutron Number (N):** $N = A - Z$.
 - **Isotopes:** Same Z , different A .
 - **Isotones:** Same N , different Z .
 - **Isobars:** Same A , different Z .
-

Solved Problems (Section 1.3)

1. Calculate the momentum of a neutron having de Broglie wavelength of 5 fm.

Formulas Used: $p = \frac{h}{\lambda}$

Solution:

$$h = 1.055 \times 10^{-21} \text{ MeV s}$$

$$\lambda = 5 \text{ fm} = 5 \times 10^{-15} \text{ m}$$

$$p = \frac{1.055 \times 10^{-21}}{5 \times 10^{-15}} = 0.211 \times 10^{-6} \text{ MeV s/m}$$

1.4 Quantitative Facts About Nucleus

1.4.1 Size

Empirical nuclear radius formula based on distance of closest approach:

$$r = r_0 A^{1/3}$$

where $r_0 = 1.2 \times 10^{-15} \text{ m} = 1.2 \text{ fm}$.

1.4.2 Mass

Nuclear masses are expressed in terms of Atomic Mass Unit (amu).

$$1 \text{ amu} = 1.660538 \times 10^{-27} \text{ kg}$$

Mass of proton = 1.0072764668 amu, Mass of neutron = 1.0086649157 amu.

1.4.3 Density

Density of a substance is defined as mass per unit volume. The nuclear density is independent of the mass number A . This can be proved mathematically as follows:

Mass of nucleus with A nucleons (M) = $A \times m$ kg (where m is the mass of one nucleon)

Volume of a nucleus with A nucleons (V) = $\frac{4}{3}\pi r^3 = \frac{4}{3}\pi(r_0 A^{1/3})^3 = \frac{4}{3}\pi r_0^3 A$

Therefore, density $\rho = \frac{M}{V} = \frac{A \times m}{\frac{4}{3}\pi r_0^3 A} = \frac{m}{\frac{4}{3}\pi r_0^3}$

Because A cancels out in the numerator and denominator, substituting for m and r_0 gives a constant value:

$$\text{Nuclear density} = 2.294 \times 10^{17} \text{ kg/m}^3$$

Thus, nuclear density is constant for all nuclei and is incredibly large (comparable only to the density of a neutron star).

1.4.4 Energy

Relation between eV, Joules, and amu via $E = mc^2$:

$$\begin{aligned} 1 \text{ eV} &= 1.602 \times 10^{-19} \text{ J} \\ 1 \text{ amu} &= 931.47 \text{ MeV} \end{aligned}$$

1.4.5 Charge

The charge on a nucleus carrying Z protons is Ze units.

Solved Problems (Section 1.4)

1. Estimate the A value and identify the nucleus if its radius is given to be 3.46 fm.

Formula Used: $r = r_0 A^{1/3}$

Solution:

$$\begin{aligned} A^{1/3} &= \frac{r}{r_0} = \frac{3.46}{1.2} = 2.883 \\ A &= (2.883)^3 = 23.97 \approx 24 \end{aligned}$$

The nucleus is ^{24}Mg .

1.5 Binding Energy

1.5.1 Mass Defect

The mass of the nucleus is always less than the sum of the masses of all nucleons present. The difference is the mass defect (Δm):

$$\Delta m = ZM_H + (A - Z)M_n - M(Z, N)$$

where M_H is the mass of a hydrogen atom, M_n is the mass of a neutron, and $M(Z, N)$ is the mass of the neutral atom.

1.5.2 Packing Fraction

Introduced by Aston for nuclear stability:

$$f = \frac{\text{Atomic mass} - \text{Mass number}}{\text{Mass number}} = \frac{\Delta m}{A}$$

1.5.3 Fusion and Fission

Heavy nuclei undergo fission, and light nuclei undergo fusion to reduce their packing fraction and increase stability, thereby releasing energy.

1.5.4 Binding Energy per Nucleon

To form an atom from requisite number of electrons, protons, and neutrons, some amount of energy is required. This energy comes from the mass defect. The energy that keeps the nucleons together in a bound state is known as the binding energy of the nucleus.

$$BE = [ZM_H + (A - Z)M_n - M(A, Z)] \times 931.47 \text{ MeV}$$

The binding energy per nucleon is given by $\frac{BE}{A}$ in MeV/nucleon. When this value is plotted against mass number A , several key conclusions can be drawn:

Placeholder for Figure 1.4: Binding energy per nucleon (in units of MeV) versus mass number A .

Figure 1: Binding energy per nucleon curve showcasing stability peaks and the broad maximum around $A = 50$.

1. Barring a few exceptions like ${}^4\text{He}$, ${}^{12}\text{C}$, ${}^{16}\text{O}$, etc., the values of binding energy per nucleon lie on a smooth continuous curve.
2. When mass number is small (i.e., $A < 12$), the binding energy per nucleon is relatively less and it rises rapidly with increasing A .

3. Around $A = 50$, there is a flat maximum where binding energy per nucleon is approximately 8.8 MeV. It slowly drops down to 8.4 MeV at $A = 140$. The average value between $A = 50$ and $A = 140$ is close to 8.5 MeV.
 4. Above $A = 140$, the binding energy per nucleon starts decreasing significantly, and at $A = 238$, its value is 7.6 MeV. It reduces further as A increases.
 5. There are sharp peaks for ${}^4_2\text{He}$, ${}^8_4\text{Be}$, ${}^{12}_6\text{C}$, and ${}^{16}_8\text{O}$ nuclei. This indicates that these specific configurations are much more stable than their immediate neighboring nuclei.
 6. **Fusion Explanation:** If we take two lighter nuclei (say ${}^2_1\text{H}$ with binding energy ~ 1.1 MeV/n) and fuse them together to form ${}^4_2\text{He}$ (binding energy ~ 7 MeV/n), there is a massive gain in total binding energy. This indicates that the fusion of lighter nuclei is highly energetically feasible.
 7. **Fission Explanation:** If we take a heavy nucleus say $A = 240$ (binding energy ~ 7.6 MeV/n) and break it into two lighter nuclei of $A \sim 120$ (binding energy ~ 8.2 MeV/n), there is again a net gain in binding energy. Therefore, the fission of heavy nuclei is energetically feasible.
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Solved Problems (Section 1.5)

1. Calculate the binding energy of α -particle and express it in MeV and joules.

Formulas Used: $BE = (Zm_p + Nm_n - m_{He}) \text{ amu}$

Solution:

$$BE = (2 \times 1.00758 + 2 \times 1.00897 - 4.0028) \text{ amu} = 0.0293 \text{ amu}$$

$$BE = 0.0293 \times 931 \text{ MeV} = 27.29 \text{ MeV}$$

$$BE = 0.0293 \times 1.49239 \times 10^{-10} \text{ J} = 4.37 \times 10^{-12} \text{ J}$$

1.6 Nuclear Angular Momentum

- **Principal (n):** $n = v + l$ in non-Coulomb fields.
- **Orbital (l):** Positive integers. Magnitude $\sqrt{l(l+1)}\hbar$.
- **Spin (s):** $1/2$ for nucleons. Magnitude $\sqrt{s(s+1)}\hbar$.
- **Total Angular Momentum Quantum Number (j):** $j = l + s$. Magnitude $\sqrt{j(j+1)}\hbar$.

1.6.1 Total Angular Momentum (L-S and j-j coupling)

L-S coupling: Orbital angular momenta sum to L ($\bar{L} = \sum \bar{l}_i$), spins to S ($\bar{S} = \sum \bar{s}_i$), and total is $J = L + S$.

j-j coupling: Individual l_i and s_i sum to j_i ($\bar{j}_i = \bar{l}_i + \bar{s}_i$), total is $J = \sum j_i$.

Solved Problems (Section 1.6)

1. One nucleon in p state ($l_1 = 1$) and another in d state ($l_2 = 2$). Find orbital angular momenta L .

Formula Used: Vector addition rules, L values between $|l_1 - l_2|$ and $l_1 + l_2$.

Solution: L values are between $|1 - 2|$ to $1 + 2$, which gives 1, 2, or 3.

1.7 Nature of Nuclear Forces

The forces known in the beginning of the twentieth century—gravitational and electromagnetic Coulomb forces—could not account for the stability of the nucleus. Protons carry positive charge, making the electrostatic force repulsive between them. The gravitational force of attraction between two nucleons is extremely weak and cannot explain binding energy. This implies a new force, the nuclear force, must exist to bind the nucleus together.

Important characteristics of nuclear forces are:

- 1. Nuclear Forces are Charge-Independent:** The force between $n-n$ is the same as that between $p-p$ and between $n-p$. For example, the binding energies of mirror nuclei like ${}^3_1\text{H}$ and ${}^3_2\text{He}$ confirm that the types of forces are equal in magnitude.
- 2. Nuclear Forces are Very Strong in Nature:** They are much stronger than electromagnetic and gravitational forces. They are stronger by a factor of 137 from the electromagnetic force and stronger by a factor of 10^{40} from gravitational forces.
- 3. Nuclear Forces have Short Range:** Nuclear forces are short-range forces and the range is of the order of nuclear dimensions. If the range of the nuclear forces is larger than the dimensions of the nucleus, they would have definitely affected molecular interactions.
- 4. Nuclear Forces are Repulsive at Very Short Distances:** If this was not so, the nucleons inside the nucleus would collapse to the size of nuclear force range. It becomes repulsive at distances ~ 0.5 fm.
- 5. Nuclear Forces are Saturated:** The range of nuclear forces is limited. Saturation implies that their interaction is only with the nearest neighbours. The effect of saturation property is that the binding energy is directly proportional to A rather than A^2 .
- 6. Nuclear Forces are Non-Central:** Electrostatic and gravitational forces act along the line joining the two particles while nuclear forces depend not only on the distance between the nucleons but also on the angle that their spins make with the line joining the two nucleons.

7. **Nuclear Forces are Spin and Angular Momentum-Dependent:** Two nucleons having parallel spin have stronger force between them than those having anti-parallel spin.
8. **Nuclear Forces are Velocity-Dependent:** Nuclear forces depend upon the velocity of the nucleons.
9. **Nuclear Forces are Exchange Forces:** The force is fundamentally produced by the exchange of a force carrier particle between the nucleons.

End-of-Chapter Questions

Short Answer Type

1. What are thermal neutrons?
2. Discuss Nuclear density.
3. Discuss why the Uncertainty principle forbids electrons in the nucleus.
4. Give any two evidences of the neutron-proton model of the nucleus.
5. Does the nucleus of the atom have sharp boundaries? How is the mass distributed in the nucleus?
6. Show that nuclear density is the same for all nuclei.
7. Why does the number of neutrons tend to exceed the number of protons in stable nuclei?
8. What are the reasons for assuming the spin of electrons?
9. Why does nuclear radius depend on mass number? Calculate its value for ^{27}Al .
10. What is average binding energy and discuss its variation with A ?
11. What do you mean by charge independence of nuclear force?
12. Explain the terms: Binding energy, Mass defect.
13. Find a relation between electron volt and amu.
14. Give one reason as to why electrons cannot exist in the nucleus.
15. Why is the density of the nucleus so large?

Long Answer Type

1. Name the forces existing between nucleons. Explain their characteristics.
2. Explain the terms: mass defect, binding energy and amu. Discuss how binding energy varies with A .
3. What is the proton-neutron hypothesis? Give a reason for the acceptance of this hypothesis. Why do electrons not exist in the nucleus?
4. Explain the terms: mass defect and binding energy of the nucleus. Give the method of calculation of binding energy for a nucleus. Also give the main features of the binding energy per nucleon versus mass number graph.
5. Explain, giving two reasons, why the electron-proton hypothesis is not acceptable, while the neutron-proton hypothesis is.
6. Explain the non-existence of electrons inside the nucleus on the basis of wave mechanical considerations.

7. Enumerate the salient features of nuclear forces. How are they different from gravitational and electromagnetic forces?
8. Explain the relationship of nuclear mass with nuclear size. Show how binding energy changes with mass number.
9. What is binding energy? Explain the stability of the nucleus on the basis of this concept.
10. Define binding energy and explain what one can learn from the variation of binding energy with A .

Unsolved Numerical Problems

1. Calculate the de Broglie wavelength for a proton having energy 150 MeV. [Ans. 0.596 fm]
2. Calculate the ratio of de Broglie wavelengths of a proton and electron having energy equal to 150 MeV. [Ans. 0.462]
3. Calculate the ratio of momenta of a proton and neutron having the same de Broglie wavelength. [Ans. 0.999]
4. Find the number of neutrons present in a ${}_{92}^{238}\text{U}$ nucleus. [Ans. 146]
5. Find the number of electrons present on a ${}_{92}^{238}\text{U}$ atom. [Ans. 92]
6. Thermal neutrons have an average kinetic energy $\frac{3}{2}kT$, where k is Boltzmann's constant and T is room temperature, 300 K. (i) What is the average energy in eV of a thermal neutron? (ii) What is the corresponding de Broglie wavelength? [Ans. (i) 0.038 eV, (ii) 1.46×10^{-10} m]
7. Find the distance of closest approach, when (i) 5 MeV protons are bombarded on ${}_{13}^{27}\text{Al}$. (ii) 5 MeV deuterons are bombarded on ${}_{13}^{27}\text{Al}$. [Ans. (i) 3.5×10^{-15} m, (ii) 3.5×10^{-15} m]
8. Find the distance of closest approach, when (i) 50 MeV protons are bombarded on ${}_{82}^{208}\text{Pb}$. (ii) 50 MeV α are bombarded on ${}_{82}^{208}\text{Pb}$. [Ans. (i) 2.87×10^{-17} m, (ii) 1.9×10^{-13} m]
9. What is the energy equivalent to the mass of an electron and a proton? [Ans. 0.511 MeV, 939.3 MeV]
10. The density of nuclear matter is 2.294×10^{17} kg/m³. Estimate the energy equivalent to 1 Å³ of nuclear matter. [Ans. 20.646 kJ]
11. Find the binding energy of ${}_{10}^{20}\text{Ne}$ and of ${}_{35}^{79}\text{Br}$. Given $m_p = 1.007825$ amu, $m_n = 1.008665$ amu, $m_{Ne} = 19.992436$ amu and $m_{Br} = 78.918336$ amu. [Ans. 160.64 MeV, 686.29 MeV]